647510 **BSc6MatC601x** **Seat No :________**

B.Sc. Semester - 6 (*CBCS*) Examination
March/April-2026 (NEW COURSE)
Graph Theory and Complex Analysis-2 (Core (New))

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Answer any two (10)

(1) Define Graph. Prove that number of vertices of odd degrees in a graph is always even.

(2) Define Euler graph. State and prove necessary and sufficient condition for a graph to be an Euler graph.

(3) Explain *Konigsberg Bridge* problem.

Que.1(B) Answer any one (04)

(1) In usual notations, prove that for any two graphs G_1 & G_2, $G_1 \oplus G_2 = G_1 \cup G_2$ and for a subgraph g of a graph G, $G \oplus g = G - g$.

(2) Prove that in simple graph with *n* vertices and *k* components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.

Que.2(A) Answer any two (10)

(1) State and prove Euler's formula for planar graphs.

(2) G is a (4, 4) graph where V = {a, b, c, d}, E = {e_1, e_2, e_3, e_4}, $\varphi(e_1) = ab, \varphi(e_2) = bc,\ \varphi(e_3) = cd, \varphi(e_4) = bd$. Find Dim W_Γ, base of W_Γ, n(W_Γ) and W_Γ, also find Dim W_S, base of W_S, n(W_S) and W_S.

(3) Define planar graph. Prove that $K_{3,3}$ and K_5 are non planar graphs.

Que.2(B) Answer any one (04)

(1) Define Incidence matrix, also state its properties.

(2) In usual notations for any simple, connected planar graph, prove that $e \geq \frac{3}{2}f$ and $e \leq 3n-6$.

Que.3(A) Answer any two (10)

(1) Prove that every Mobious transformation maps circles OR straight lines into circles OR straight lines.

(2) Discuss the transformation w= sin z.

(3) Obtain the transformation of real axis of z – plane , under the mapping $w = \frac{2+iz}{i+4z}$.

Que.3(B) Answer any one (04)

(1) Obtain the transformation of region $x \geq C_1$ of z – plane under the mapping $w = \frac{1}{z}$

(2) Find the mobious mapping which transforms the vertices of Δ ABC where A(1+i), B(-i), C(2-i) of z – plane into the vertices A'(0), B'(1), C'(i) of $\Delta\, A'B'C'$ of w – plane

Que.4(A) Answer any two (10)

(1) State and prove Taylor's infinite series for an analytic function.

(2) State and prove Laurent's infinite series for an analytic function.

(3) Expand $\frac{1}{z-a}$ in Laurent's series for |a| < |z| and |a| < 1 and if $z = e^{i\theta}$ then prove that $\sum_{n=0}^{\infty} a^n \cos n\theta = \frac{a\cos\theta - a^2}{1-2a\cos\theta + a^2}$ and $\sum_{n=1}^{\infty} a^n \sin n\theta = \frac{a\sin\theta}{1-2a\cos\theta + a^2}$.

Que.4(B) Answer any one (04)

(1) Expand $\frac{z}{(z-1)(z-3)}$ in Laurent's series for $0 < | z-1 | < 2$.

(2) Expand $\frac{z-1}{z^2}$ in Taylor's series for $z = 1$.

Que.5(A) Answer any two (10)

(1) State and prove Cauchy's residue theorem.

(2) Prove by using Residue Theorem $\int_c \frac{zdz}{(z^4-1)} = 0$.

(3) Prove by using Residue Theorem $\int_0^\infty \frac{cosxdx}{x^2+1} = \frac{\pi}{2e}$.

Que.5(B) Answer any one (04)

(1) Define: Residue and derive formula for finding Residue of f(z) at simple pole z_0

(2) Using Residue theorem prove that $\int_0^{2\pi} \frac{d\theta}{5+4cos\theta} = \frac{2\pi}{3}$.

****